

P425/1
PURE MATHEMATICS
Paper 1
April 2025
3 Hours

SET 5

GRANT K EXAMS 2025
Uganda Advanced Certificate of Education
PURE MATHEMATICS
PAPER 1
3 Hours

INSTRUCTIONS:

- Attempt all the **eight** questions in section **A** and any **five** from section **B**.
- All working must be clearly shown.
- Mathematical tables with a list of formulae and squared paper are provided.
- Silent, simple non programmable scientific calculators and a list of formulae may be used.
- State the degree of accuracy at the end of each answer using **CAL** for calculator and **TAB** for tables.

SECTION A (40 Marks)
Attempt ALL questions in this section.

1. Solve the inequality $\frac{1+x}{4+x} \geq \frac{5-2x}{x}$ (5marks)
2. Evaluate $\int_3^4 \frac{1}{x^2-3x+2} dx$ (5marks)
3. Solve the equation $2\tan\theta + \sin 2\theta \sec\theta = 1 + \sec\theta$ for $0 \leq \theta \leq 2\pi$. (5marks)

4. The line $5x - 2y + 8 = 0$ is a tangent to the circle with centre at $(-2, 3)$. Find the equation of the circle. (5marks)
5. Expand $(25 - 2x)^{\frac{1}{2}}$ in ascending powers of x up to the term in x^3 . Hence by taking $x=1$, obtain the value of $\sqrt{23}$ correct to four significant figures. (5marks)
6. If $y = e^{2x} \sin 2x$, show that $\frac{d^2 y}{dx^2} = 8(e^{2x} \cos^2 x - 1)$. (5marks)
7. The position vectors of the points P and Q are $3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ respectively. Find the acute angle between PQ and the line;
 $1 - x = \frac{y-3}{2} = \frac{4-x}{2}$ (5marks)
8. Solve the differential equation, $\left(\frac{dy}{dx}\right)^3 = e^{(x-3y)}$. Given that $y(6) = 0$. (5marks)

SECTION B (60MARKS)

Attempt ONLY 5 questions in this section.

9. a) Show that;
 $(xy) = \frac{1}{2}x + \frac{1}{2}y$. Hence or otherwise, solve the simultaneous equations.
 $(xy) = \frac{7}{2}$
 $\frac{x}{y} = -8$ (7marks)
- b) Solve the equation $2^{(2+2x)} + 3 \cdot 2^x - 1 = 0$. (5marks)
10. a) Find x , if $\cos^{-1}\left(\frac{x}{2}\right) = \frac{5\pi}{6}$. (5marks)
- b) Express $5\sin\theta + 12\cos\theta$ in the form $r \sin(\theta + a)$, giving the value of r and a , hence find $5\sin\theta + 12\cos\theta = 7$. (7marks)

11. a) Differentiate with respect to x .
 i) x^x
 ii) $\tan^{-1}\left(\frac{1-x}{1+x}\right)$, simplify your answers (8marks)
 b) if $y = e^{4x} \cos 3x$, show that $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 25y = 0$. (4marks)
12. a) Show that the line $\frac{x-2}{2} = \frac{y-2}{-1} = \frac{z-3}{3}$ and the plane $\underline{r} \cdot (4 - 1 - 3) = 4$ are parallel and find the perpendicular distance of the line from the plane. (6marks)
 b) Find the equation of the plane passing through the origin and parallel to the lines' $\frac{x+2}{3} = \frac{y-1}{4} = \frac{z+1}{5}$ and $\frac{x-3}{4} = \frac{y-2}{-5} = \frac{z+1}{1}$. (6marks)
13. a) Solve the differential equation $x^2 \frac{dy}{dx} = y(y + x)$; Given that $y(4) = 6$. (4marks)
 b) A certain game park was found to have 100 lions. Given that the lions die at a rate proportional to the number of lions present and the initial death rate is 5 lions per year.
 i) Form a differential equation and solve it.
 ii) How many lions will be in the park after six years? (8marks)
14. a) Given that $Z = \cos \cos \phi + i \sin \phi$, where $\phi \neq \pi$, show that $\frac{2}{1+z} = 1 - i \tan\left(\frac{1}{2}\phi\right)$. (6marks)
 b) The polynomial $P(z) = z^4 - 3z^3 + 7z^2 + 21z - 26$ has $2 + 3i$ as one of the roots. Find the other three roots of the equation $P(z) = 0$. (6marks)
15. a) Work out $\int \frac{dx}{e^x - 1} dx$. (5marks)
 b) The area bounded by the curve $y = x(x - 4)$, and the x -axis is rotated about the x -axis through a $\frac{1}{2}$ -turn. Find the volume of the solid generated. (7marks)

16. a) Find an equation of the circle that passes through the points.
 $A(-1, 4)$, $B(2, 5)$ and $C(0, 1)$. (5marks)
- b) The line $x + y = c$ is a tangent to the circle
 $x^2 + y^2 - 4y + 2 = 0$. Find the coordinates of the points of
contact of the tangent for each value of C . (7marks)

END